

A Stochastic Gradient Method for PDE Constrained Optimization under Uncertainty

Caroline Geiersbach

University of Vienna,
Oskar-Morgenstern-Platz 1

1090 Vienna, Austria. Email: caroline.geiersbach@univie.ac.at

Models incorporating uncertain inputs, such as random forces or material parameters, have been of increasing interest in PDE constrained optimization. In this talk, we focus on the efficient numerical solution to minimizing the expectation of a random function subject to an elliptic partial differential equation with random coefficients and box constraints. The problem can be expressed in the form

$$\min_{u \in \mathcal{U}^{\text{ad}}} j(u) = \mathbb{E}[J(u, \xi)],$$

where \mathcal{U}^{ad} is a nonempty, closed and convex subset of a Hilbert or a Banach space.

The approach we take is based on a classical method proposed by Robbins and Monro in 1951. In place of true gradient $\nabla j(u)$, a stochastic gradient $G(u, \xi) \approx \nabla j(u)$ is chosen based on one or more samples from a known probability distribution. Feasibility is maintained by performing a projection at each iteration; the method takes the form

$$u_{n+1} = \pi_{\mathcal{U}^{\text{ad}}}(u_n - \tau_n G(u_n, \xi_n)).$$

In the application of this method to PDE constrained optimization under uncertainty, new challenges arise. We observe the discretization error made by approximating the gradient using finite elements. Analyzing the interplay between numerical and stochastic error, we develop a mesh refinement strategy coupled with decreasing step sizes. We also demonstrate the approach on a more challenging problem, namely a model structural topology optimization problem using a phase field representation for shapes.

Joint work with: Winnifried Wollner, Luise Blank