Workshop on "Phase Field Models and Free Boundary Problems"

organised by Helmut Abels and Balázs Kovács

supported by University of Regensburg, Faculty of Mathematics

and DFG-Research Training Group 2339 – *IntComSin*

Regensburg, 22–24 June, 2023

On the occasion of the **60th birthday of Harald Garcke** we will enjoy interesting talks on phase field models and free boundary problems, meeting old friends and some social programme around this one-day workshop and colloquium the day before.

On Thursday the colloquium talk will take place in **lecture hall H31**. The talks on Friday will take place in the seminar room **M104**, while coffee breaks will be in the common room **M201**. On Friday we will have lunch together at the university canteen.

Preliminary Programme

Thursday, June 22, 2023

Thursday	
16:30 – 17:00	*Coffee break*
17:00 – 18:00	Colloquium talk by Björn Stinner
19:00	*Dinner at Leerer Beutel*

Friday, June 23, 2023

We will have 45 minute talks (including discussion) by the following speakers.

Friday	
10:15 – 11:00	Robert Nürnberg
11:00 – 11:30	*Coffee break*
11:30 – 12:15	Barbara Niethammer
12:15 – 14:00	*Lunch break*
14:00 – 14:45	Michael Hinze
14:45 – 15:30	Klaus Deckelnick
15:30 – 16:00	*Coffee break*
16:00 – 16:45	Matthias Röger
16:45 – 18:00	*Open discussions*
18:30	*Workshop dinner at Bischofshof am Dom*

Saturday, June 24, 2023

For those wo wish to stay on Saturday, as a social programme we will enjoy a tour to the picturesque nearby village, Kallmünz in the morning. There is the possibility to go on hikes of various difficulty, and also to enjoy the city, or a local café. Afterwards we will have lunch together in a restaurant, such that in the afternoon everyone has enough time to travel home. The departure from Regensburg to Kallmünz is planed for 9:30 and lunch around 12:00–12:30.

Colloquium talk – Thursday

Björn Stinner (University of Warwick)

Lecture hall H31, Thursday, 17:00–18:00

Title: **Phase Fields, Moving Meshes, and Triple Junctions**

Abstract: Problems with free or moving boundaries feature in various applications, such as crystal growth, multi-phase flow, or evolving cell boundaries. Modelling and computational approaches may be based on directly tracking the boundary, for instance, by a parametrisation of a hypersurface. Alternatively, one may attempt to implicitly capture it in terms of fields defined in the ambient domain. In any case, identifying underlying energy functionals and related variational structures is desirable as a fundament for a mathematical analysis and the development of stable computational methods. We give an overview of some contributions by Harald Garcke to the field. Using moving meshes to track an evolving interface generally leads to degenerate mesh elements, and we present an idea to address this issue in a way that naturally fits into the variational framework. However, the focus will be on phase field approaches. We will provide some insight on how the methodology works and showcase some significant results, and illustrate it with some applications.

Detailed workshop programme – Friday

Michael Hinze and Klaus Deckelnick (University of Koblenz–Landau and University of Magdeburg)

Friday, 14:00 – 14:45 and 14:45 – 15:30

Title: **Lipschitz methods in PDE constrained shape optimization**
Part 1: Concept and (fluid mechanical) applications (Michael Hinze)
Part 2: Numerical analysis (Klaus Deckelnick)

Abstract: We present a general framework for PDE constrained shape optimisation based on the method of mappings, where we use Lipschitz mappings for the domain variations. We propose steepest descent and Newton-like minimisation algorithms for the numerical solution of the respective shape optimisation problems. Our work is built upon previous work of the authors in (Deckelnick, Herbert, and Hinze, ESAIM: COCV 28 (2022)), where a Lipschitz framework for star-shaped domains is proposed. To illustrate our approach we present in the first part a selection of PDE constrained shape optimisation problems and compare our findings to results from so far classical Hilbert space methods and recent p-approximations. In the second part we propose a finite element approximation of the shape optimization problem and numerically analyse the convergence of a steepest descent method. We underpin our numerical analysis findings with numerical experiments which support our analytical findings.

This is joint work with Philip Herbert (St. Andrew), with Peter Marvin Müller and Thomas Rung (both TU Hamburg), and with Martin Siebenborn.

Barbara Niethammer (University of Bonn)

Friday, 11:30 – 12:15

Title: **On a free boundary problem in a model for cell polarization**

Abstract: We investigate a model for cell polarization under external stimulus where a diffusion equation in the inner cell is coupled to reaction diffusion equations on the cell membrane. In certain scaling limits we rigorously derive generalized obstacle type problems. For these limit systems we prove global stability of steady states and characterize the parameter regime for the onset of polarization. We also discuss some aspects of regularity in time of the free boundary. The talk is based on joint work with Anna Logioti (Bonn) Matthias Röger (TU Dortmund) and Juan Velázquez (U Bonn)

Robert Nürnberg (University of Trento)

Friday, 10:15 – 11:00

Title: **A generalized DeTurck trick for anisotropic curve shortening flow**

Abstract: We consider the motion of parametric curves by anisotropic curve shortening flow. We propose a strictly parabolic system of PDEs for the position vector whose solution satisfies the corresponding evolution law. This system is discretized by linear finite elements and optimal H^1 error bounds are proved in the semidiscrete case. In addition, we consider some fully discrete schemes and prove their unconditional stability. Finally, we present several numerical experiments that demonstrate the robustness and practicality of the introduced schemes. This is joint work with Klaus Deckelnick.

Matthias Röger (University of Technology Dortmund)

Friday, 16:00 – 16:45

Title: **An alternative phase field approximation of the Willmore energy and mean curvature flow**

Abstract: We consider a phase field approximation of mean curvature flow based on multiple microscopic mechanisms. For the associated dissipation functional we prove the Gamma convergence to the Willmore energy. In a generalized formulation we obtain the long-time convergence of the phase field evolution to (a rescaled) mean curvature flow. (This is joint work with Sascha Knüttel, TU Dortmund)