Global bifurcation of capillary-gravity water waves with overhanging profiles and arbitrary vorticity

While the research on water waves modeled by Euler's equations has a long history, mainly in the last two decades traveling periodic rotational waves have been constructed with mathematical rigor by means of bifurcation theorems. In this talk, I will present a new reformulation of this traveling periodic water wave problem in two dimensions and in the presence of surface tension, gravity, and a flat bed. Using conformal mappings and a new Babenko-type reformulation of Bernoulli's equation, the problem is equivalently cast into the form ``identity + compact'', which is amenable for Rabinowitz' global bifurcation theorem. The main advantages of this new reformulation are that no simplifying restrictions on the geometry of the surface profile and no simplifying assumptions on the vorticity distribution (and thus no assumptions regarding the absence of stagnation points) have to be made. Within the scope of this new formulation, local and global solution curves, bifurcating from laminar flows with a flat surface, are constructed. Moreover, I will further discuss the condition for local bifurcation and the possible alternatives for ``endpoints'' of the global curve. This is joint work with Erik Wahlén.