Shape-optimization based focusing of quasilinear acoustic waves

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Focused ultrasound waves of high intensity are an essential non-invasive tool in many medical procedures, such as kidney stone treatments by lithotripsy or treatments of certain types of cancer. In these applications, focusing of sound is achieved geometrically, often with an acoustic lens. Such setups can naturally benefit from shape optimization. However, the use of high-power ultrasound sources enhances nonlinear effects in wave propagation. In this talk, we will give an overview of typical challenges arising in analysis and numerics of nonlinear acoustics and discuss shape optimization subject to nonlinear sound waves.

To model the propagation of sound through fluid and acoustic lens, we use Westervelt's quasi-linear wave equation with piecewise constant coefficients. We formulate the shape optimization problem by employing an L^2 tracking-type cost functional. This allows to reach desired high-pressure values in the focal region of, for example, a kidney stone and low-pressure values around it. We follow the optimize first, then discretize approach, and rely on rigorous shape calculus. A gradient-based optimization algorithm is employed to find a locally optimal shape. The implementation is realized within the framework of isogeometric analysis, where geometry representation, as well as the approximation space for the pressure field, share the same spline basis functions. The theoretical considerations are illustrated by numerical experiments in a 2D setting.

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