Error estimates for reaction-diffusion processes through thin heterogeneous layers in case of homogenized equations with interface diffusion

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In this talk, we consider a nonlinear reaction–diffusion equation in a microscopic domain consisting of two bulk-domains, which are separated by a thin layer with a periodic heterogeneous structure. The size of the heterogeneities and thickness of the layer are of order ϵ, where the parameter ϵ is small compared to the length scale of the whole domain. The numerical computation of the solution to this type of problems faces a high complexity. Therefore, we construct approximations of the microscopic solution which can be calculated with less numerical effort, and prove error estimates between the approximation and the microscopic solution with respect to the scaling parameter ϵ.

In the singular limit $\epsilon \to 0$, when the thin layer reduces to an interface Σ separating two bulk domains, a macroscopic model with effective interface conditions across Σ is obtained. For our specific scaling, the macroscopic solution is continuous across the interface Σ and the difference of the normal fluxes from the bulk solutions is given by a reaction-diffusion equation for the macroscopic solution on Σ. Our approximations are obtained by adding corrector terms to the macroscopic solution, which take into account the oscillations in the thin layer and the coupling conditions between the layer and the bulk domains. To validate these approximations, we prove error estimates with respect to ϵ. Our approximations are constructed in two steps leading to error estimates of order $\epsilon^{\frac{1}{2}}$ and ϵ in the H^1-norm.