Hydrodynamic limits of the Boltzmann equation

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In this talk I will discuss hydrodynamic limits of the Boltzmann equations which lead to the equations of fluid dynamics. We will start by nondimensionalizing the equations which leads to the identification of the Knudsen number Kn (= fraction between the mean free path of particles between collisions and the observation length scale) as the crucial scaling parameter for hydrodynamic limits. Another important parameter is the Mach number Ma (= fraction between bulk velocity and the typical particle velocity (the latter is the speed of sound)).

We will formally show that in the limit $Kn \to 0$ with Ma fixed, one obtains the compressible Euler equations for ideal monoatomic gases, and that the compressible Navier-Stokes equations is a higher order correction in terms of Kn.

We then discuss the limit $Kn = Ma = \varepsilon \to 0$ which gives rise to Leray solutions of the incompressible Navier-Stokes equations. I will discuss some ingredients of the rigorous proof.