Mathematical analysis of phase separation models

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Phase separation in a binary liquid (e.g. oil and vinegar) is a phenomenon which can be described as a competition between an entropy mixing effect and a demixing effect due to the internal energy. Typical mathematical models are given by the so-called Cahn-Hilliard (CH) equation or by the conserved Allen-Cahn (AC) equation with singular potential. The Cahn-Hilliard (CH) equation can be considered as to be local or nonlocal, according to the type of interactions (short range or long range) we take into account. These equations govern the evolution of the relative concentration of one component and conserve the total mass. Moreover, one can consider a multicomponent mixture, so that more than two chemical species can come into play in phase separation phenomena. In this talk, I will first present the phase separation phenomenon through some of its recent and unexpected applications in cell biology, showing its growing and growing importance in many fields of mathematical modelling. Then I will introduce some of the aforementioned equations, like CH and AC equations on bounded domains, trying to give an insight of the most recent results about the mathematical analysis concerning these equations. In conclusion, I will introduce the instantaneous strict separation property, meaning that each (weak) solution, which is not a pure phase initially, stays uniformly away from the pure phases from any positive time on. I will show how this property is essential to obtain higher-order regularity as well as to study the long-time behavior of solutions and I will present some recent advancements in showing the validity of this property in case of 3D AC and nonlocal CH with singular potential.