Evolving microstructures in multiscale problems

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Phase transition processes and swelling of porous media are examples of moving boundary problems where the geometry is allowed to evolve in time and where microscopic effects influence the macroscopic properties of the system. As a consequence of the changes in geometry, the resulting mathematical problems are generally highly non-linear and complex and the scale separation adds further difficulties.
With the Hanzawa transformation, we present a popular technique for moving boundary problems and discuss how it can be applied in a multiscale setting. In the simplified case of uniform cell evolutions, we use this transformation to show well-posedness of the moving boundary problem and to conduct a limit analysis with respect to the scale parameter.
We also present a numerical scheme involving a pre-computing technique used to solve the limit problem efficiently.