Are turbulent flows computable ?

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There are at least three basic (partly interlinked) problems for the numerical simulation of turbulent incompressible Navier-Stokes flows:

- the "curse of resolution" for direct numerical simulation (DNS) with strong dependence on the Reynolds number
- regularity and intermittency of the solution connected with the Onsager hypothesis [2],
- the zeroth law of turbulence in the inviscid limit which may lead to anomalous diffusion.

These problems call for good solutions w.r.t. computational ressources.

H(div)-conforming dGFEM ensure pointwise divergence-free discrete velocities together with pressure-robustness, convection semi-robustness and structure preservation. They allow a weak imposition of the tangential velocities and act as an implicit large-eddy-simulation (ILES).

In particular, we discuss results for the 3D-Taylor-Green vortex in the inviscid case where Fehn et al. [1] showed grid-convergence to a weak dissipative solution. Mathematically, it is a challenge to prove convergence of the discretization to a weak dissipative Euler solution. Such questions have been addressed for the compressible Euler problems e.g. by Kuzmin et al. [3]. We sketch problems of an extension to the incompressible case.

Moreover, we present results for turbulent wall-bounded flows and discuss problems of extensions to the inviscid incompressible case.

References

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