Dimension reduction via Gamma-convergence of non-convex energies, and the quantitative immersability of Riemannian metrics.

ABSTRACT: In this talk, we present results that relate the following three contexts:

(i) Given a Riemannian metric G we study the problem of finding the infimum of the averaged pointwise deficit of an immersion from being an orientation-preserving (equi-dimensional) isometric immersion of G. We are also interested in discovering the structure and regularity of the optimal immersion. Questions of this type can be seen as a quantitative variation of the very classical topics in Differential Geometry: existence, rigidity and flexibility of isometric immersions.

(ii) Given a family of non-convex energies E^h, parametrized by a three-dimensional plate's thickness h, we are interested in finding the scaling laws of the infima of E^h, in terms of the powers of h, as h goes to 0. Subsequently, we want to derive the Gamma-limits (variational limits) of E^h and reveal the asymptotic structure of the minimizing sequences. This topic is classical in the Calculus of Variations and may also be seen as a variation of the study of nonlinear elasticity.

Combining (i) and (ii), we first show how the deficit mentioned in (i) can be measured by the energies E^h in (ii), and perform the full scaling analysis of E^h, in the context of dimension reduction. Then, we derive the Gamma-limits of $h^{-2n}E^h$ for all powers n. We show the energy quantization, in the sense that the even powers 2n of h are the only possible ones (all of them are also attained). For each n, we identify conditions for the validity of the corresponding scaling, in terms of the vanishing of Riemann curvatures of G up to appropriate orders, and in terms of the singular behavior of the minimizing immersions.

We will further explain how the above problems are linked to:

(iii) The description of elastic materials exhibiting residual stress at free equilibria. Examples of such structures and their actuations include: plastically strained sheets; specifically engineered swelling or shrinking gels; growing tissues, and atomically thin graphene layers. Our results and methods display the interaction of nonlinear pdes, geometry and mechanics of materials in the prediction of patterns and shape formation.

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