Sharp Interface Limit of mass-conserving Stokes/Allen-Cahn System

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In this talk, we explore the sharp interface limits of mass-conserving Allen-Cahn and Stokes/ Allen-Cahn systems, which model phase separation in binary fluid mixtures. Using matched asymptotic expansions and rigorous error estimates, we derive the boundary behavior as the interface thickness parameter ε approaches zero. For the Allen-Cahn equation, we show convergence to a volume-preserving mean curvature flow. We then couple it to the Stokes equation. Key contributions include detailed asymptotic constructions, careful remainder estimates, and convergence proofs based on spectral analysis of the linearized operators. We use a novel Ansatz for the highest order term in the asymptotic expansion and then estimate its difference with the real solution using a refined spectral estimate of the solution using a refined spectral estimate of the linearized Allen-Cahn operator near the approximate solution. This work provides a mathematically rigorous basis for understanding the macroscopic dynamics of phase-separating fluids from their microscopic descriptions and the connection between the diffuse interface model and the sharp interface limit.