Structure-preserving parametric finite element method for surface diffusion based on Lagrange multiplier approaches

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In this talk, we discuss a novel formulation for parametric finite element methods to simulate surface diffusion for both closed curves and surfaces. Several high-order temporal discretizations are proposed based on this new formulation. To ensure that the numerical methods preserve geometric structures of surface diffusion, our formulation incorporates two scalar Lagrange multipliers and two evolution equations involving the perimeter/surface-area and area/volume, respectively. By discretizing the spatial variable using piecewise linear finite elements and the temporal variable using the backward differentiation formulae method, we develop first-order and second-order temporal schemes that effectively preserve the structure at a fully discrete level. These new schemes are implicit and can be efficiently solved using Newton's method. Extensive numerical experiments demonstrate that our methods achieve the desired temporal accuracy, while simultaneously preserving the geometric structure of the surface diffusion.