Adaptive methods for stochastic PDEs
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Abstract: The construction of adaptive methods to approximate solutions of deterministic PDEs is an established part in the numerical analysis of PDEs: many of them exploit a posteriori error estimates - where numerical errors are bounded by computable terms only - to steer automatic space(-time) mesh refinements/coarsenings to accomplish the aim to accurately represent approximate solutions on meshes as coarse as possible.

A corresponding strategy is relevant as well for stochastic PDEs, where the numerical analysis is much less developed. Immediate questions appear: what type of meshes should be given preference ('deterministic or random'), or what are relevant errors ('weak or strong errors, or errors in law') on which to base automatic space-time remeshing?

I start this series of talks with an a priori error analysis of the stochastic heat equation, repeating the practically relevant variational solution concept for this SPDE, of stability estimates for its (time-implicit) spatio-temporal discretization, and of Kolmogorov's equation. The main part then proposes three conceptionally different adaptive methods, whose relevancy depends on the answer to my question above:

(i) this method focuses on a strong approximation, which is based on residual-based estimators.
(ii) here we aim for a weak approximation, which uses the related Kolmogorov's equation.
(iii) this method is based on a comparison of subsequent empirical measures, which are constructed via data-dependent partitionings of the high-dimensional finite-element space.

The lectures base on joint works with A.K. Majee (IIT Delhi), with C. Schellnegger (earlier: U Tuebingen), and F. Merle (U Tuebingen).