Shape and critical point evolution under planar curvature-driven flows

Planar curvature-driven flows describe the evolution of curves by prescribing the velocity in the normal direction in terms of the curvature. The simplest of such flows is the curve shortening flow (the velocity is proportional to the curvature), first proposed by Firey to model the abrasion of colliding stones of equal size. Firey's model was broadly generalized both mathematically (Andrews) and geophysically (Bloore), resulting in what we refer to as the Andrews-Bloore flow. Existing numerical techniques, such as the level-set method, fail to capture the fine geometrical properties of the evolving curves. In particular, the time evolution of geophysical shape descriptors is of key interest to geophysicists, and one of the most important shape descriptors is the number N of critical points if the curve is interpreted as a scalar distance from a fixed point or the center of mass. While the evolution N(t) is of central importance in geomorphology, it is particularly sensitive to numerical errors as it relies on derivatives. In my talk, I will describe a front-tracking algorithm using a polynomial representation of the curve, which we developed with the main goal to investigate N(t) in the Andrews-Bloore flows.