Abstracts:

Existence and regularity results for the stochastic Cahn-Hilliard equation

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We present some global existence and regularity results for the stochastic Cahn-Hilliard equation. First of all, we introduce the equation as one of the main representatives of the theory of diffuse-interface modelling and we describe the abstract mathematical setting. Secondly, we show existence of solutions in a weak sense using techniques from monotone analysis and compactness results. Finally, we prove that under refined assumptions on the data the solutions inherit further regularity, and a natural variational formulation of the problem can be written.

Mixed-Dimensional PDEs: From Functional Analysis to Discretization Methods

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Mixed-dimensional models arise in cases where thin structures are represented as manifolds of lower dimension within the computational domain. Examples include fractures and faults in the subsurface, membranes and shells in biological models, as well as mechanical reinforcements in engineering applications. These models are typically governed by coupled equations defined on manifolds of different dimensions, which we refer to as mixed-dimensional partial differential equations.
As a leading example, we use the mixed-dimensional representation of fracture networks on which we consider Darcy flow, thermal conduction, and linear elasticity. In order to describe the governing equations, we introduce semi-discrete differential operators that map between manifolds of different dimension. The mixed-dimensional problem is then considered as a whole, and several fundamental tools are required to facilitate its analysis.
The focus of this work lies on the underlying structure of mixed-dimensional PDEs and we present extensions of well-established results from functional analysis to the mixed-dimensional setting. Sobolev spaces are constructed using suitable inner products and differential operators, resulting in the formation of a de Rham complex. The constructed de Rham complex is then used to form stable discretization methods for mixed-dimensional PDEs. In order to respect physical conservation laws locally, we use the mixed finite element method as a primary example and show how stable finite element spaces can directly be chosen in each dimension using finite element exterior calculus.