

Regularization by noise for the stochastic transport equation

Prof. Dr. Lisa Beck (University of Augsburg)

Abstract:

We discuss several aspects of regularity and uniqueness for weak (L^∞ -) solutions to the (deterministic and stochastic) transport equation

$$du = b \cdot D u dt + \sigma Du \circ dW_t.$$

Here, b is a vector field (the drift), u is the unknown, σ is a real number, W_t is a Brownian motion, and the stochastic term is interpreted in the Stratonovich sense.

For the deterministic equation ($\sigma = 0$) it is well-known that multiple solutions may exist and that solutions may blow up from smooth initial data in finite time if the drift is not regular enough. For the stochastic equation ($\sigma \neq 0$) instead, it turns out that a suitable integrability condition (known from fluid dynamics as the Ladyzhenskaya–Prodi–Serrin condition) on the drift is sufficient to prevent the formation of non-uniqueness and of singularities. After a short review of some techniques for the deterministic equation we explain how this regularization phenomenon, namely the conservation of Sobolev regularity of the initial data and the restoration of uniqueness, is obtained by means of PDE techniques (as opposed to stochastic characteristics). The results presented in this talk are part of a joint project with F. Flandoli, M. Gubinelli and M. Maurelli.

Boundary value problems for the one-dimensional Helfrich equation

PD Dr. Klaus Deckelnick (University of Magdeburg)

Abstract:

We consider the Euler-Lagrange equation for the one-dimensional Helfrich functional in the class of graphs subject to either Dirichlet or Navier boundary conditions. Combining variational methods and invariance properties of the functional we derive existence and uniqueness results. Furthermore, we examine qualitative properties of solutions as well as their dependence on boundary data in the Dirichlet case. This is joint work with Anna Dall'Acqua (Ulm).